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Numerical Simulation of Fluid-Structure Interaction Problems on Hybrid Meshes with AMG

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Numerical simulation of fluid-structure interaction problems on hybrid meshes with algebraic multigrid methods

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Abstract. Fluid-structure interaction problems arise in many application fields such as flows around elastic structures or blood flow problems in arteries. The method presented in this paper for solving such a problem is based on a reduction to an equation at the interface, involving the so-called Steklov-Poincaré operators. This interface equation is solved by a Newton-like iteration. One step of the Newton-like iteration requires the solution of several decoupled linear sub-problems in the structure and the fluid domains. These sub-problems are spatially discretized by a finite element method on hybrid meshes. For the time discretization the implicit Euler method is used. The discretized equations are solved by algebraic multigrid methods.

1 Problem setting of the fluid-structure interaction

1.1 Geometrical description and the ALE mapping

Let Ω_0 denote the initial domain at a time $t = 0$ consisting of the structure and fluid sub-domains Ω_0^{s} and Ω_0^{f}, respectively. The domain $\Omega(t)$ at a time t is composed of the deformable structure sub-domain $\Omega^{\mathrm{s}}(t)$ and the fluid sub-domain $\Omega^{\mathrm{f}}(t)$. The corresponding interface $\Gamma(t)$ is evolving from the initial interface Γ_0. The evolution of $\Omega(t)$ is obtained by an injective mapping, the so-called ALE mapping (Fig. 1.):

$$x : \Omega_0 \times R^+ \to R^3. \tag{1}$$

The position of a point $x_0 \in \Omega_0^{\mathrm{s}}$ at a time t is given by the mapping for the structure domain

$$x_t^{\mathrm{s}} : \Omega_0^{\mathrm{s}} \to \Omega^{\mathrm{s}}(t), \tag{2}$$

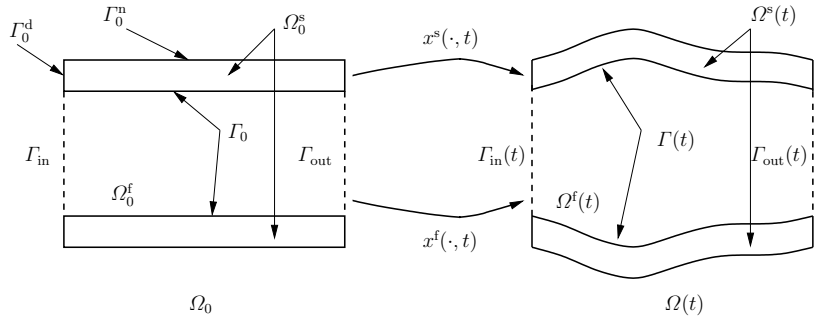


Fig. 1. ALE mapping

given by $x_t^{\mathrm{s}}(x_0) \equiv x^{\mathrm{s}}(x_0,t) = x(x_0,t) = x_0 + d^{\mathrm{s}}(x_0,t)$ for $x_0 \in \Omega_0^{\mathrm{s}}$, where $d^{\mathrm{s}}(x_0,t)$ denotes the displacement $d^{\mathrm{s}}(x_0,t)$ of the structure domain at a time t.

Correspondingly, the position of any point $x_0 \in \Omega_0^{\mathrm{f}}$ at a time t is given by the mapping for the fluid domain

$$x_t^{\mathrm{f}} : \Omega_0^{\mathrm{f}} \to \Omega^{\mathrm{f}}(t) \tag{3}$$

given by $x_t^{\mathrm{f}}(x_0) \equiv x^{\mathrm{f}}(x_0,t) = x(x_0,t) = x_0 + d^{\mathrm{f}}(x_0,t)$ for $x_0 \in \Omega_0^{\mathrm{f}}$, where $d^{\mathrm{f}}(x_0,t)$ denotes the displacement in fluid domain. It is defined as an extension of the structure displacement d^{s} at the interface Γ_0:

$$d^{\mathrm{f}} = \mathrm{Ext}(\, d^{\mathrm{s}}|_{\Gamma_0}), \tag{4}$$

e.g. the harmonic extension, given by:

$$\begin{cases} -\Delta d^{\mathrm{f}} = 0 & \text{in } \Omega_0^{\mathrm{f}}, \\ d^{\mathrm{f}} = d^{\mathrm{s}} & \text{on } \Gamma_0, \\ d^{\mathrm{f}} = 0 \quad \text{on } \Gamma_{\mathrm{in}}(t) \cup \Gamma_{\mathrm{out}}(t). \end{cases} \tag{5}$$

Furthermore, we introduce the domain velocities by

$$w^{\mathrm{s}}(x_0,t) := \frac{\partial d^{\mathrm{s}}}{\partial t}(x_0,t) = \frac{\partial x^{\mathrm{s}}}{\partial t}(x_0,t)$$

and

$$w^{\mathrm{f}}(x_0,t) := \frac{\partial d^{\mathrm{f}}}{\partial t}(x_0,t) = \frac{\partial x^{\mathrm{f}}}{\partial t}(x_0,t)$$

for the structure and the fluid domain respectively.

1.2 The physical model in strong form

We describe the interface conditions which have to be satisfied for this coupled problem.

Interface conditions. At the interface between the structure and the fluid domain we assume no-slip conditions:

$$d^{\mathrm{f}}\big|_{\Gamma_0} = d^{\mathrm{s}}|_{\Gamma_0} \tag{6}$$

and the equilibrium of normal stresses:

$$(\sigma_{\mathrm{f}} n_{\mathrm{f}}) \circ x_t^{\mathrm{f}} + \sigma_{\mathrm{s}} n_{\mathrm{s}} = 0, \tag{7}$$

where σ_{s} is the first Piola-Kirchoff stress tensor, σ_{f} is the Cauchy stress tensor, n_{f} and n_{s} are the outward normals of $\Omega^{\mathrm{f}}(t)$ and Ω_0^{s}, respectively.

Structure and fluid sub-problems. With prescribed Dirichlet data λ for the displacement at the interface Γ_0, we compute the Neumann data $\sigma_{\mathrm{s}} n_{\mathrm{s}}$ at the interface Γ_0 by solving the structure problem

$$\begin{cases} \rho_{\mathrm{s}} \dfrac{\partial^2 d^{\mathrm{s}}}{\partial t^2} - \mathrm{div}(\sigma_{\mathrm{s}}(d^{\mathrm{s}})) = 0 & \text{in } \Omega_0^{\mathrm{s}}, \\ \sigma_{\mathrm{s}}(d^{\mathrm{s}}) n_{\mathrm{s}} = 0 & \text{on } \Gamma_0^{\mathrm{n}}, \\ d^{\mathrm{s}} = 0 & \text{on } \Gamma_0^{\mathrm{d}}, \\ d^{\mathrm{s}} = \lambda & \text{on } \Gamma_0, \end{cases} \tag{8}$$

where ρ_{s} is the density. We will concentrate on a linear Saint-Venant Kirchoff elastic model, i.e. $\sigma_{\mathrm{s}}(d^{\mathrm{s}}) = 2\mu^l \varepsilon(d^{\mathrm{s}}) + \lambda^l \mathrm{div}(d^{\mathrm{s}}) I$ with $\varepsilon(d^{\mathrm{s}}) = \frac{\nabla d^{\mathrm{s}} + (\nabla d^{\mathrm{s}})^T}{2}$ and the Lamé constants λ^l, μ^l.

We introduce S_{s} as the Dirichlet-to-Neumann mapping

$$S_{\mathrm{s}} : H^{1/2}(\Gamma_0) \to H^{-1/2}(\Gamma_0), \tag{9}$$

given by $S_{\mathrm{s}}(\lambda) = \sigma_{\mathrm{s}}(d^{\mathrm{s}}) n_{\mathrm{s}}$, with an appropriate function space $H^{1/2}(\Gamma_0)$ and its dual space $H^{-1/2}(\Gamma_0)$.

Let $u(x,t)$ denote the Eulerian velocity of the fluid. The ALE time derivative of $u(x,t)$ is introduced in order to overcome the difficulty for evaluating the time derivative of velocity $u(x,t)$ under the Eulerian framework in a moving domain. Let $x \in \Omega^{\mathrm{f}}(t)$ with $x_0 = {x_t^{\mathrm{f}}}^{-1}(x)$, then the ALE time derivative is given by

$$\left.\frac{\partial u}{\partial t}\right|_{x_0}(x,t) = \frac{d}{dt}\left(u(x_t^{\mathrm{f}}(x_0), t)\right). \tag{10}$$

Analogously, with prescribed Dirichlet data $\frac{\partial \lambda}{\partial t}$ for the velocity at the interface Γ_0, we compute the Neumann data $(\sigma_{\mathrm{f}} n_{\mathrm{f}}) \circ x_t^{\mathrm{f}}$ at the interface Γ_0 by solving the fluid problem

$$\begin{cases} \rho_{\mathrm{f}} \left.\dfrac{\partial u}{\partial t}\right|_{x_0} + \rho_{\mathrm{f}}\left((u - w^{\mathrm{f}}) \cdot \nabla\right) u - 2\mu \mathrm{div}\varepsilon(u) + \nabla p = 0 & \text{in } \Omega^{\mathrm{f}}(t), \\ \mathrm{div} u = 0 & \text{in } \Omega^{\mathrm{f}}(t), \\ \sigma_{\mathrm{f}}(u,p) n_{\mathrm{f}} = g_{\mathrm{in}} & \text{on } \Gamma_{\mathrm{in}}(t), \\ \sigma_{\mathrm{f}}(u,p) n_{\mathrm{f}} = 0 & \text{on } \Gamma_{\mathrm{out}}(t), \\ u \circ x_t^{\mathrm{f}} = \dfrac{\partial \lambda}{\partial t} & \text{on } \Gamma_0, \end{cases} \tag{11}$$

where ρ_{f} is the density of the fluid, μ its dynamic viscosity, the stress tensor $\sigma_{\mathrm{f}}(u,p) = -pI + 2\mu\varepsilon(u)$, the pressure p, the strain tensor $\varepsilon(u) = \frac{\nabla u + (\nabla u)^{\mathrm{T}}}{2}$.

In a similar way as before, we introduce the Dirichlet to Neumann mapping

$$S_{\mathrm{f}} : H^{1/2}(\Gamma_0) \to H^{-1/2}(\Gamma_0), \tag{12}$$

given by $S_{\mathrm{f}}(\lambda) = (\sigma_{\mathrm{f}}(u,p)n_{\mathrm{f}}) \circ x_t^{\mathrm{f}}$.

With these notations the equilibrium condition (7) can be written as

$$S(\lambda) := S_{\mathrm{f}}(\lambda) + S_{\mathrm{s}}(\lambda) = 0, \tag{13}$$

which is the so-called Steklov-Poincaré equation and will be solved by iterative method in Section 2.

1.3 Weak formulations

For the weak formulation we need the function spaces $V^{\mathrm{s}} = [H^1(\Omega_0^{\mathrm{s}})]^3$, $V_0^{\mathrm{s}} = \{v^{\mathrm{s}} \in V^{\mathrm{s}} | v^{\mathrm{s}} = 0 \text{ on } \Gamma_0^{\mathrm{d}} \bigcup \Gamma_0\}$, and $V_g^{\mathrm{s}} = \{v^{\mathrm{s}} \in V^{\mathrm{s}} | v^{\mathrm{s}} = \lambda(t) \text{ on } \Gamma_0\}$ for the structure. For the fluid, we define $D^{\mathrm{f}} = [H^1(\Omega_0^{\mathrm{f}})]^3$, $D_0^{\mathrm{f}} = \{d \in D^{\mathrm{f}} | d = 0 \text{ on } \Gamma_0\}$, $D_g^{\mathrm{f}}(t) = \{d \in D^{\mathrm{f}} | d = \lambda(t) \text{ on } \Gamma_0\}$, $V^{\mathrm{f}}(t) = \{v^{\mathrm{f}} | v^{\mathrm{f}} \circ x_t^{\mathrm{f}} \in H^1(\Omega_0^{\mathrm{f}})^3\}$, $V_0^{\mathrm{f}}(t) = \{v^{\mathrm{f}} \in V^{\mathrm{f}}(t) | v^{\mathrm{f}} \circ x_t^{\mathrm{f}} = 0 \text{ on } \Gamma_0\}$, $V_g^{\mathrm{f}}(t) = \{v^{\mathrm{f}} \in V_0^{\mathrm{f}} | v^{\mathrm{f}} \circ x_t^{\mathrm{f}} = w^{\mathrm{f}} \circ x_t^{\mathrm{f}} \text{ on } \Gamma_0\}$, and $Q^{\mathrm{f}}(t) = \{q^{\mathrm{f}} | q^{\mathrm{f}} \circ x_t^{\mathrm{f}} \in L^2(\Omega_0^{\mathrm{f}})\}$, where $H^1(\Omega_0^{\mathrm{s}})$ and $H^1(\Omega_0^{\mathrm{f}})$ denote the standard Sobolev spaces.

Then we obtain:

The weak form of the structure problem: Find $d^{\mathrm{s}} \in V_g^{\mathrm{s}}$ such that for all $v^{\mathrm{s}} \in V_0^{\mathrm{s}}$,

$$a(d^{\mathrm{s}}, v^{\mathrm{s}}) = 0 \tag{14}$$

with

$$a(d^{\mathrm{s}}, v^{\mathrm{s}}) = \int_{\Omega_0^{\mathrm{s}}} \rho_{\mathrm{s}} \frac{\partial^2 d^{\mathrm{s}}}{\partial t^2} \cdot v^{\mathrm{s}} dx_0 + \int_{\Omega_0^{\mathrm{s}}} [\lambda^l \mathrm{div} d^{\mathrm{s}} \mathrm{div} v^{\mathrm{s}} + 2\mu^l \varepsilon(d^{\mathrm{s}}) : \varepsilon(v^{\mathrm{s}})] dx_0. \tag{15}$$

The weak form for the harmonic extension: Find $d^{\mathrm{f}} \in D_g^{\mathrm{f}}(t)$ such that for all $\phi \in D_0^{\mathrm{f}}$,

$$a(d^{\mathrm{f}}, \phi) = 0 \tag{16}$$

with

$$a(d^{\mathrm{f}}, \phi) = \int_{\Omega_0^{\mathrm{f}}} \nabla d^{\mathrm{f}} : \nabla \phi dx_0. \tag{17}$$

The computational fluid domain $\Omega^{\mathrm{f}}(t)$ is then given by

$$\Omega^{\mathrm{f}}(t) = \Omega_0^{\mathrm{f}} + d^{\mathrm{f}}. \tag{18}$$

The weak form of the fluid problem: Find $(u,p) \in V_g^{\mathrm{f}}(t) \times Q^{\mathrm{f}}(t)$ such that for all $(v^{\mathrm{f}}, q^{\mathrm{f}}) \in V_0^{\mathrm{f}}(t) \times Q^{\mathrm{f}}(t)$,

$$\begin{cases} a(u, v^{\mathrm{f}}) + b_1(v^{\mathrm{f}}, p) = \langle F^{\mathrm{f}}, v^{\mathrm{f}} \rangle, \\ b_2(u, q^{\mathrm{f}}) - c(p, q^{\mathrm{f}}) = \langle G^{\mathrm{f}}, q^{\mathrm{f}} \rangle, \end{cases} \tag{19}$$

where

$$\begin{cases} a(u, v^{\mathrm{f}}) = \dfrac{\mathrm{d}}{\mathrm{d}t} \displaystyle\int_{\Omega^{\mathrm{f}}(t)} \rho_{\mathrm{f}} u \cdot v^{\mathrm{f}} dx - \int_{\Omega^{\mathrm{f}}(t)} \mathrm{div} w^{\mathrm{f}} \rho_{\mathrm{f}} u \cdot v^{\mathrm{f}} dx \\ \qquad\qquad + \displaystyle\int_{\Omega^{\mathrm{f}}(t)} \rho_{\mathrm{f}} \left((u - w^{\mathrm{f}}) \cdot \nabla \right) u \cdot v^{\mathrm{f}} dx + 2\mu \int_{\Omega^{\mathrm{f}}(t)} \varepsilon(u) : \varepsilon(v^{\mathrm{f}}) dx, \\ b_1(v^{\mathrm{f}}, p) = b_2(v^{\mathrm{f}}, p) = - \displaystyle\int_{\Omega^{\mathrm{f}}(t)} p \ \mathrm{div} v^{\mathrm{f}} dx, \\ c(p, q^{\mathrm{f}}) = 0, \quad \langle F^{\mathrm{f}}, v^{\mathrm{f}} \rangle = \displaystyle\int_{\Gamma_{\mathrm{in}}(t)} g_{\mathrm{in}} \cdot v^{\mathrm{f}} ds, \quad \langle G^{\mathrm{f}}, q^{\mathrm{f}} \rangle = 0. \end{cases} \tag{20}$$

The weak form of the equilibrium condition: Find $\lambda(t) \in H^{1/2}(\Gamma_0)$ such that, for all $v^{\mathrm{f}} \times v^{\mathrm{s}} \in V^{\mathrm{f}}(t) \times V^{\mathrm{s}}$ such that

$$\langle S_{\mathrm{f}}(\lambda), v^{\mathrm{f}} \rangle_{\Gamma(t)} + \langle S_{\mathrm{s}}(\lambda), v^{\mathrm{s}} \rangle_{\Gamma_0} = 0. \tag{21}$$

where $v^{\mathrm{f}} \circ x_t^{\mathrm{f}} = v^{\mathrm{s}}$ on Γ_0 and $\langle ., . \rangle$ denotes the corresponding dual product.

1.4 Discretization

The spatial discretization was done by a finite element method on a hybrid mesh consisting of tetrahedra, hexahedra, pyramids and prisms. These elements, see the first line of Fig. 2, are splitted into pure tetrahedral elements by introducing artificial points at the volume and face centers, see the second line of Fig. 2. We then construct the finite elements based on pure tetrahedral meshes. The introduced additional degrees of freedom are locally eliminated by the mean value approximation, for details [5].

A fully implicit time discretization is used for the structure problem. The fluid problem is discretized in time by a semi-implicit method.

2 Iterative methods for the interface equation

We apply a preconditioned Richardson method to (13): given λ^0, for $k \geq 0$,

$$\lambda^{k+1} = \lambda^k + \omega^k P_k^{-1} \left(-S_{\mathrm{s}}(\lambda^k) - S_{\mathrm{f}}(\lambda^k) \right), \tag{22}$$

with the relaxation parameter ω^k and a proper preconditioner P_k.

The Newton algorithm applied to (13) is obtained by using iteration (22) and choosing the preconditioner at step k (see [1]): $P_k = S_{\mathrm{s}}'(\lambda^k) + S_{\mathrm{f}}'(\lambda^k)$. In

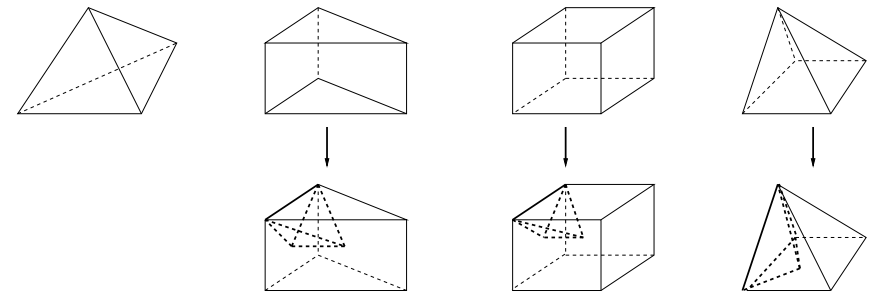


Fig. 2. Splitting of hybrid elements into tetrahedral elements.

our computation, we instead use an approximation of the full tangent operators $\hat{P}_k \approx S_{\mathrm{s}}^{'}(\lambda^k) + S_{\mathrm{f}}^{'}(\lambda^k)$ which includes the Fréchet derivative for the structure operator S_{s} and the classical Fréchet derivative part which does not take into account the shape change for the fluid operator S_{f}.

In each step of the iterative method a problem of the form

$$\hat{P}_k \mu^k = - \left(S_{\mathrm{s}}(\lambda^k) + S_{\mathrm{f}}(\lambda^k) \right) \tag{23}$$

has to solved. For this we used a preconditioned GMRES method with preconditioner $S_{\mathrm{s}}^{'}(\lambda^k)$ (see [1]).

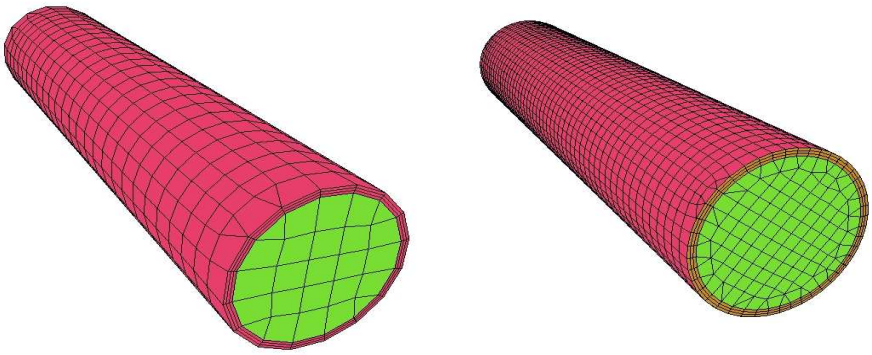
Summarizing, the method can be described as follows: for $k \geq 0$

1. update the residual $S_{\mathrm{s}}(\lambda^k) + S_{\mathrm{f}}(\lambda^k)$ by solving the structure and fluid sub-problems,
2. solve the tangent problem (23) via GMRES method,
3. update the displacement λ^{k+1}, if not accurate enough, go to step 1.

Note that Step 1 can be parallelized due to the independence of the sub-problems for given interface boundary conditions. Step 2 requires solving the linearized structure and fluid problems several times during the GMRES iteration, for details we refer to [5]. The algebraic multigrid method (AMG) is used for the fluid and structure sub-problems, see [3] and [4].

3 Numerical results

We simulate a pressure wave in a cylinder of length 5 cm and radius 5 mm at rest. The thickness of the structure is 0.5 mm. The structure is considered linear and clamped at both the inlet and outlet. The fluid viscosity is set to $\mu = 0.035$, the Lamé constants to $\mu^l = 1.15 \times 10^6$ and $\lambda^l = 1.73 \times 10^6$, the density to $\rho^{\mathrm{f}} = 1.0$ and $\rho^s = 1.2$. The fluid and structure are initially at rest and a pressure



(a) coarse mesh (b) fine mesh

Fig. 3. Fine and coarse meshes for simulations.

of 1.332×10^4 dyn/cm^2 is set on the inlet for a time period of 3 ms. Two meshes (Fig.3.) are used for simulations:

A relative error reduction by a factor of 10^{-5} is achieved in 2-3 outer iterations. Each of these iterations requires 6-8 GMRES iterations for a relative error reduction by a factor of 10^{-5}. For solving the structure problem, about 10 preconditioned conjugate gradient iterations with AMG preconditioning are needed for a relative error reduction by a factor of 10^{-8}, for the fluid problem about 5 AMG iterations for a relative error reduction by a factor of 10^{-8}. Almost the same numbers of iterations were observed for the coarse and the fine mesh.

For all simulations, we use a time step size of $\delta t = 1$ ms and run the simulation until time $t = 25$ ms. For visualization purposes the deformation is amplified by a factor of 12. Fig.4 shows the pressure wave propagation on the fine mesh at different time levels.

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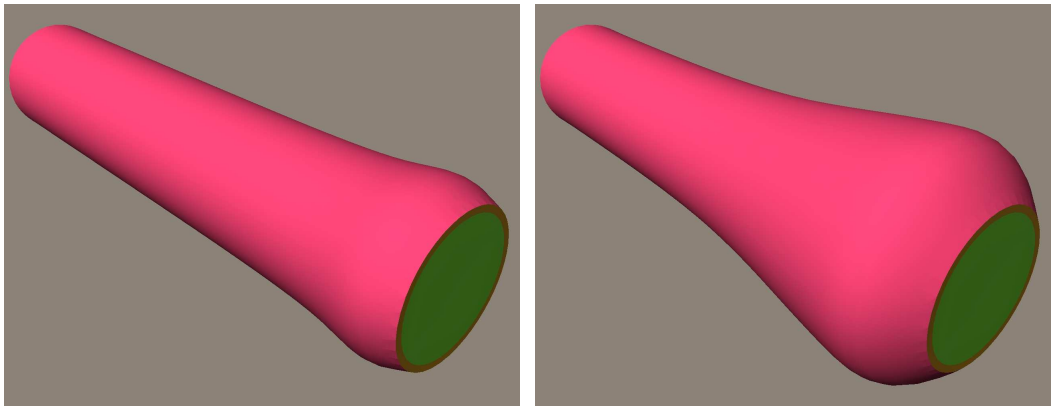
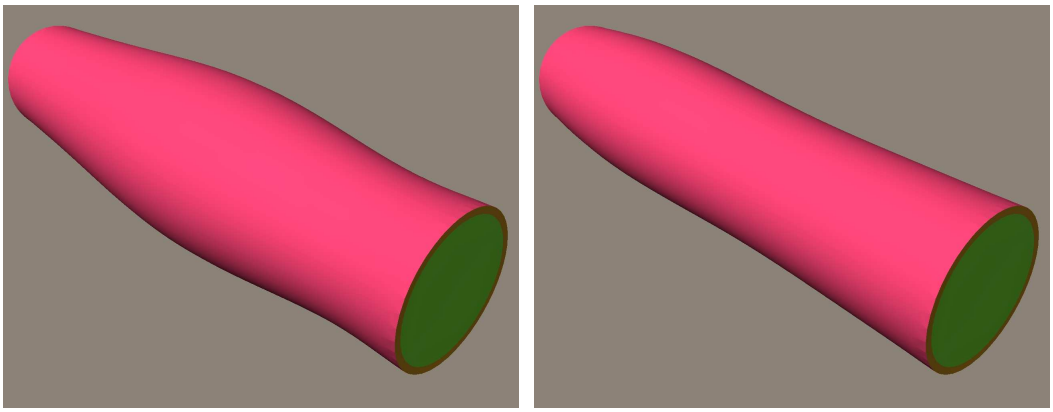
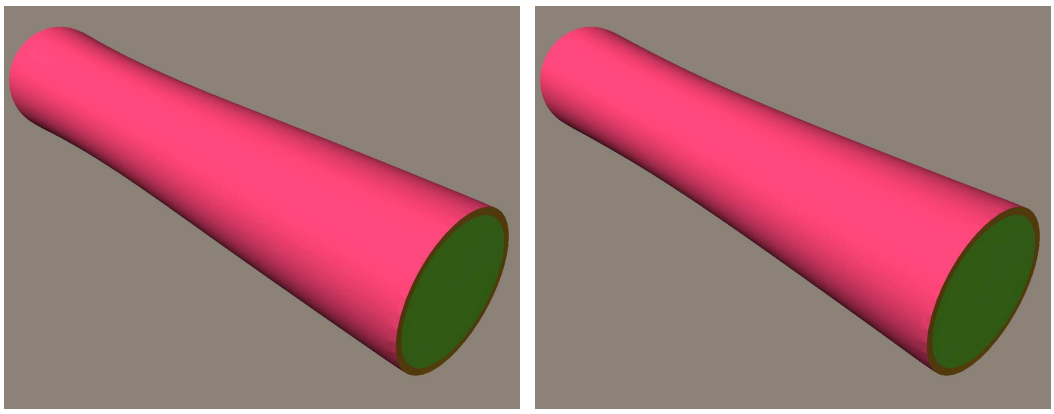


(a) $t = 1$ ms (b) $t = 5$ ms



(c) $t = 10$ ms (d) $t = 15$ ms



(e) $t = 20$ ms (f) $t = 25$ ms

Fig. 4. Simulation results at time $t = 1$ ms (upper left), $t = 5$ ms (upper right), $t = 10$ ms (middle left) and $t = 15$ ms (middle right), $t = 20$ ms (lower left) and $t = 25$ ms (lower right).

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