10th CMAPT Workshop
Computational Mathematics and Approximation Theory

03 – 07 July 2023, Sozopol, Bulgaria

organized by
Institute of Mathematics and Informatics,
Bulgarian Academy of Sciences, Bulgaria
Institute of Numerical Mathematcs (NuMa),
Johannes Kepler University Linz, Austria,
and
Research Institute of Symbolic Computation (RISC),
Johannes Kepler University Linz, Austria

Schedule

Arrival day	July 2
Departure day	July 8
Excursion day	July 5
Lectures	July, 3, 4, 6, 7, The House of the Bulgarian Red Cross, Cherno more str. 1

Program

Monday, July 3, 2023

14:00 – 14:15	Registration	
14:15 – 14:30	Opening	
14:30 – 15:15	Michael Mandlmayr	Normal cone computation with CAD (Cylindric Algebraic Decomposition)
15:15 – 16:00	Irina Georgieva	Computation of polynomial and rational approximations in complex domains

Tuesday, July 4, 2023

09:15 – 10:00	Geno Nikolov	Turan inequality for ultraspherical polynomials revisited
10:00 – 10:45	Stefan Takacs	Locking free Isogeometric Analysis of linearized elasticity using reduced quadrature?
10:45 – 11:15	Coffee break	
11:15 – 12:00	Ivan Gadjev	Extremal function and sequence for Hardy Inequalities in Hilbert Spaces
12:00 – 14:00	Lunch break	
14:00 – 17:00	Discussions in groups	
19:00	Official dinner	

Wednesday, July 5, 2023

09:30 – 16:00	Excursion	

Thursday, July 6, 2023

09:15 – 10:00	Stefan Tyoler	Efficient computation of a spline basis for adaptive multi-patch discretizations
10:00 – 10:45	Rumen Uluchev	Asymptotically sharp estimates for the best constant in a weighted Hardy inequality
10:45 – 11:15	Coffee break	
11:15 – 12:00	Georgi Bazlyankov	Differential equation models of the different types of Multiple Sclerosis
12:00 – 14:00	Lunch break	
14:00 – 17:00	Discussions in groups	

Friday, July 7, 2023

09:15 – 10:00	Vladimira Suvandjieva	A model for Lions-Hyenas interactions
10:00 – 10:45	Bogdan Radu	A mixed FEM for the nonlinear magnetostatic problem
10:45 – 11:15	Coffee break	
11:15 – 12:00	Anniversary talk	
12:00 – 14:00	Lunch break	
14:00 – 16:00	Discussions in groups	
16:00	Closing	

Abstracts

Differential equation models of the different types of Multiple Sclerosis

Georgi Bazlyankov

Multiple sclerosis (MS) is a neurodegenerative autoimmune disease which affects millions of people around the world. While MS has no known causes and cures, the abstract methods of mathematical modelling could help us obtain a better understanding of the pathological mechanisms of the disease and predict its progression in patient specific cases. In this talk, we consider a PDE model of the main cell types active in MS pathogenesis. The model's practical constraints are highlighted and modifications are proposed with the aim of better describing a wider range of disease types. A delay differential equation model and a model with time-varying coefficients are proposed. For the latter, a 4-stage relapse-remission cycle is proposed for types of MS with quasiperiodic behavior. A special case of MS called Baló's concentric disease allows us to demonstrate the model's practical applicability. Numerical realizations of the models are carried out in the Wolfram Mathematica software. We also address the theoretical and computational challenges when dealing with unconventional differential equations such as delay differential equations.

Extremal function and sequence for Hardy Inequalities in Hilbert Spaces

Ivan Gadjev

The behavior of the smallest possible constants $d(a,b)$ and d_n in classical Hardy inequalities

$$\int_a^b \left(\frac{1}{x}\int_a^x f(t)dt\right)^2 dx \le d(a,b) \int_a^b f^2(x)dx$$

and

$$\sum_{k=1}^{n} \Big(\frac{1}{k}\sum_{j=1}^{k} a_j\Big)^2 \le d_n \sum_{k=1}^{n} a_k^2$$

is discussed. The exact constant $d(a,b)$ and the exact rate of convergence of d_n are established and the extremal function and the "almost extremal" sequence are found. Also, the same questions about the weighted versions of the above inequalities

$$\int_a^b \left(\int_a^x f(t)dt\right)^2 u(x)\, dx \le d(a,b) \int_a^b f^2(x)v(x)\, dx,$$

and

$$\sum_{k=1}^{n} \Big(\sum_{j=1}^{k} a_j \Big)^2 u_k \leq d_n \sum_{k=1}^{n} a_k^2 \, v_k$$

are discussed. The important cases for weighted functions $u(x) = x^{\epsilon-2}$, $v(x) = x^{\epsilon}$ and weighted sequences $u_k = k^{\epsilon-2}$, $v_k = k^{\epsilon}$ are considered.

This study is financed by the European Union-NextGenerationEU, through the National Recovery and Resilience Plan of the Republic of Bulgaria, project No BG-RRP-2.004-0008.

Computation of polynomial and rational approximations in complex domains

Irina Georgieva, Clemens Hofreither

Motivated by applications in fractional partial differential equations, we investigate some numerical methods for computation of polynomial and rational approximations of functions in complex domains. We give an overview over classical results and then describe the basis of Faber polynomials associated with a domain as well as expansion in the Faber series. We also explore the use of the Lanczos tau-method for approximation with complex polynomials as well as with rational functions.

We present some numerical examples, where sectors lying in the complex plane are the domains of interest.

This work is supported by grant KP-06-N62/4, funded by Bulgarian National Science Fund.

Normal cone computation with CAD (Cylindric Algebraic Decomposition)

Michael Mandlmayr

In optimization many questions are based on the so called normal cone (Lagrangian). Recent developements have been made in the area of semismooth* Newton methods, where normal cones appear as a linearization tool.

Calculus rules exist but they require so-called constraint qualifications. However, for a given semialgebraic set, the basic definition can be written in a fully polynomial language and cylindric algebraic decomposition can be applied. The upside is that these methods do not require constraint qualification. However the complexity is very bad in the number of variables.

Therefore, this approach seems to be particularly interesting for product structures, with technical difficulties like complementaritiy contraints.

We will present the application to an academic complementarity constraint optimization problem and for computing coderivatives for the contact problem with Coulomb friction.

Turan inequality for ultraspherical polynomials revisited

Geno Nikolov

In my talk I discuss some old and recent results about Turan's inequality. A proof of a recent result about convexity/concavity of the normalized Turanian for ultraspherical polynomials will be presented.

A mixed FEM for the nonlinear magnetostatic problem

Bogdan Radu

We discuss numerical methods for solving nonlinear magnetostatic problems in two dimensions, which are of relevance in the simulation of electric machines. In contrast to standard approaches, the magnetic flux B is approximated by H(div)-conforming finite elements. The discretization thus leads to saddle-point systems, which we reduce to elliptic problems employing hybridization. In this setting, the Lagrange multipliers appearing in this setting correspond to the tangential component of the magnetic field intensity H. As a consequence, both traces $B\cdot n$ and $H\times n$ are available at element interfaces and can be used, e.g., for torque computation. We present numerical experiments and comparisons to existing methods.

A model for Lions-Hyenas interactions

Vladimira Suvandjieva

The motivation of this work is the study of indirect competition of resources for a particular ecosystem, consisting of four populations: lions, prey, carcasses and hyenas. First of all a simple predator-prey model is considered where lions play the role of the predator. Both cases are analysed: exponential and logistic growth of the prey. Next the model is extended with two more equations describing the carcasses and hyenas as part of the ecosystem. Two scenarios of interaction between lions and hyenas are observed. In the first case only the fact that lions can kill hyenas is considered. In the second case, on the other hand, the fact that hyenas can somehow affect lions growth rate is also considered. Finally, some numerical results are shown in each case.

Locking free Isogeometric Analysis of linearized elasticity using reduced quadrature?

Stefan Takacs

Isogeometric Analysis is a relative novel approach that allows the use of CAD geometries for numerical solution. Technically, it is a B-spline based finite element approach using a global parametrization of the computational domain. We are interested in the solution of linearized elasticity equations with isogeometric analysis. The variational formulation of the linearized elasticity problem involves a bilinear form that is elliptic and bounded. If one considers the case of (almost) incompressible materials, the conditioning of the problem degrades. This leads to poor approximation power. This phenomenon is known as locking.

It is possible to circumvent locking by setting up a saddle point formulation. Unfortunately, this significantly increases the number of degrees of freedom. For certain finite element discretizations, it is possible to again eliminate these additional degrees of freedom. The resulting discrete variational problem can be seen as a version of the original variational problem, where the original integrals are replaced by quadrature rules that do not coincide with the integrals (reduced quadrature). By such approaches, locking can be avoided. I am interested in the question if such reduced quadrature is also possible in Isogeometric Analysis. Here, the quadrature has to be a reduced quadrature (in order to avoid locking), but still good enough to maintain the superior approximation power of Isogeometric Analysis (as compared to low-order methods).

Efficient computation of a spline basis for adaptive multi-patch discretizations

Stefan Tyoler

In this talk, we propose a new approach for adaptive discretizations in Isogeometric Analysis. In order to avoid the non local refinement of tensor product discretizations in 2D or higher dimensions, we decompose the computational domain into multiple geometrically non-conforming patches. On each of these patches, we set up individual tensor product discretizations. Since we use different grid sizes on each patch we usually have non conforming but nested discretizations on the interfaces. The nesting property allows the coupling of local basis functions in a H^1 conforming way across interfaces. This also applies to T-junctions emerging from the local refinements.

We further give some insight on the computation and formation of a spline basis using a classical and a more algebraic approach and the problems that come with each of these approaches. Finally, we show some results by employing this method to a simple adaptive test problem, utilizing patchwise refinement and a residual a posteriori error estimator.

Asymptotically sharp estimates for the best constant in a weighted Hardy inequality

Rumen Uluchev

In 1925, G. H. Hardy stated and proved an integral inequality, later known as *Hardy inequality*. In 1928 he proved a weighted generalization of the inequality, which reads: If $p > 1$ and $\alpha < p - 1$, then

$$\int_0^\infty \left(\frac{1}{x} \int_0^x f(t)\, dt \right)^p x^\alpha \, dx \le \Big(\frac{p}{p-1-\alpha} \Big)^p \int\limits_0^\infty f^p(x) x^\alpha \, dx,$$

for and all nonnegative measurable functions f. Also, the constant $\big(\frac{p}{p-\alpha-1}\big)^p$ cannot be replaced by a smaller one.

Let $\alpha < 1$. We study the rate of the smallest possible constant $c_n(\alpha)$ in the weighted Hardy integral inequality

$$\int_0^\infty \left(\frac{1}{x} \int_0^x f(t)\, dt \right)^2 x^\alpha \, dx \le c_n(\alpha) \int\limits_0^\infty f^2(x) x^\alpha \, dx,$$

for functions f from the n-dimensional space

$$\mathcal{H}_n := \left\{ f \,:\, f(x) = \int_0^x f(t)\,dt = x^{-\alpha/2} e^{-x/2}\, p(x),\ p \in \mathcal{P}_n,\ p(0) = 0 \right\},$$

where $\mathcal{P}_n$ is the set of real-valued algebraic polynomials of degree at most n.

Our main result is that we prove asymptotically sharp two-sided estimates for $c_n(\alpha)$ and fix the rate of convergence as $n \to \infty$.