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Talk announcement

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An asymptotic preserving hybrid-dG method for convection-diffusion equations on pipe networks

We consider convection-diffusion equations on one-dimensional pipe networks, describing, e.g., the transport and diffusion of a contaminant solved in a fluid. In addition to boundary conditions at network boundary nodes, we need suitable coupling conditions at pipe junctions in order to guarantee conservation of mass as well as stability and well-posedness. The number and type of coupling conditions change in the limit of pure transport, which gives rise to boundary layers at pipe junctions for vanishing diffusion. On a single pipe it is well known that solutions to convection-diffusion problems converge to that of the transport problem, and we can show that a similar result also holds on networks.For the numerical approximation we propose a hybrid discontinuous Galerkin method which turns out to be especially suitable for transport dominated problems on networks when dealing with coupling conditions. It also has the property of being asymptotic preserving, i.e., is able to handle the change in number and type of coupling conditions in the singular limit. The standard error analysis leads to degenerate convergence results for vanishing diffusion due to the presence of boundary layers. Estimates for the derivatives of the solution to the singularly perturbed convection-diffusion problem are well-known on single intervals,and we show that similar estimates also hold on networks. By using this knowledge we then set up a graded mesh on the network and derive improved order optimal error estimates for the singularly perturbed problem. Numerical tests will illustrate our theoretical findings.